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CHANGE IN THE ELASTIC PARAMETERS AND STRENGTH OF ROCKS UNDER PRESSURE

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ABSTRACT: Models are used to demonstrate the effect porosity in rocks has on change in elastic parameters when rocks are subjected to high uniform pressures. It is shown that the change can be found by knowing the relationship between areas of contacts and units strain, and that this latter relationship can be found by using certain formulas and curves, both of which are included.

Computed on Models

Numerous experimental papers devoted to the study of the elastic properties of rocks under high pressure conditions have shown that there is a sharp increase in Young's modulus, and in the velocity of elastic waves under high uniform pressure, up to 1.000 to 2.000 kg/cm^2. /394* There is a reduction in the gradient at which the elastic characteristics increase with further increase in pressure. Most authors explain this behavior of rocks under high pressure conditions by the change in contacts between grains of the minerals, and by the partial closing of pores and cracks [1, 2, 3].

In order to evaluate the effect of the pore space on the elastic and strength properties of rocks, let us consider several models with different types of porosity, and let us compute their elastic and strength characteristics. If we take it that upon application of stresses to an isotropic rock contained in a shell the stresses occurring within the specimen are greater than the applied stresses, because of porosity, we can then present the unit strain for the specimen as

$$\varepsilon_x = \frac{p_x}{E_0\left(1-\frac{S_{2yz}}{S_1}\right)} - \frac{\mu_0 p_y}{E_0\left(1-\frac{S_{2xz}}{S_1}\right)} - \frac{\mu_0 p_z}{E_0\left(1-\frac{S_{2xy}}{S_1}\right)},$$

$$\varepsilon_y = -\frac{\mu_0 p_x}{E_0\left(1-\frac{S_{2yz}}{S_1}\right)} + \frac{p_y}{E_0\left(1-\frac{S_{2xz}}{S_1}\right)} - \frac{\mu_0 p_z}{E_0\left(1-\frac{S_{2xy}}{S_1}\right)}, \quad (1)$$

$$\varepsilon_z = -\frac{\mu_0 p_x}{E_0\left(1-\frac{S_{2yz}}{S_1}\right)} - \frac{\mu_0 p_y}{E_0\left(1-\frac{S_{2xz}}{S_1}\right)} + \frac{p_z}{E_0\left(1-\frac{S_{2xy}}{S_1}\right)},$$

* Numbers in the margin indicate pagination in the foreign text.

where

$\varepsilon_x, \varepsilon_y, \varepsilon_z$, are the unit strains for the body along the corresponding axes;

p_x, p_y, p_z, are the applied stresses;

S_1 is the area of the specimen;

$S_{2yz}, S_{2xz}, S_{2xy}$, are the areas of the cross sections of the pore space perpendicular to the corresponding axes;

E_0, μ_0 are Young's modulus and Poisson's ratio for the solid substance of the rocks.

As we know, rocks can have pores with many different shapes. Let us break them down conditionally into three groups; volumetric pores, or pores with all three dimensions of the same order of magnitude; slotted and microslotted pores, in which one dimension is shorter than the other two by one order of magnitude. In order to ascertain which pores have the greatest effect on change in elasticity, let us take three models of a solid body in the form of cubes with chaotically distributed pores. Let us take as our first model the body with volumetric pores only. The second model will contain intermediate pores, as well as volumetric pores. The third model will have all three types of pores. Let us consider the case when a uniform pressure, $p = p_y$, plus the additional compressive stress $p_2 = p_x - p_y$, /395 is applied to the model. Then Young's modulus and Poisson's ratio along the x axis can be expressed as

$$E_x = \frac{\Delta p_x}{\Delta \varepsilon_x}, \quad \mu_{yx} = \frac{\Delta \varepsilon_y}{\Delta \varepsilon_x}. \tag{2}$$

We will take $E_0 = 10^6$ kg/cm^2, $\mu_0 = 0.25$ as conventional in the calculations since they correspond to the Young's modulus and the Poisson's ratio for a number of rocks under high pressures.

The methodology used to make the computations was reviewed in an earlier paper [4]. It simply reduces to the necessity of first establishing the connection between the unit strain, ε, and the area of contacts between grains, S_1, S_2, or the area of the pore space, S_2. We can construct this relationship by assigning to the model definite types of pores, and by taking it that their distribution in terms of the dimensions is subordinate to a normal law. A similar relationship can, in fact, be found for a real rock. Figure 1 shows

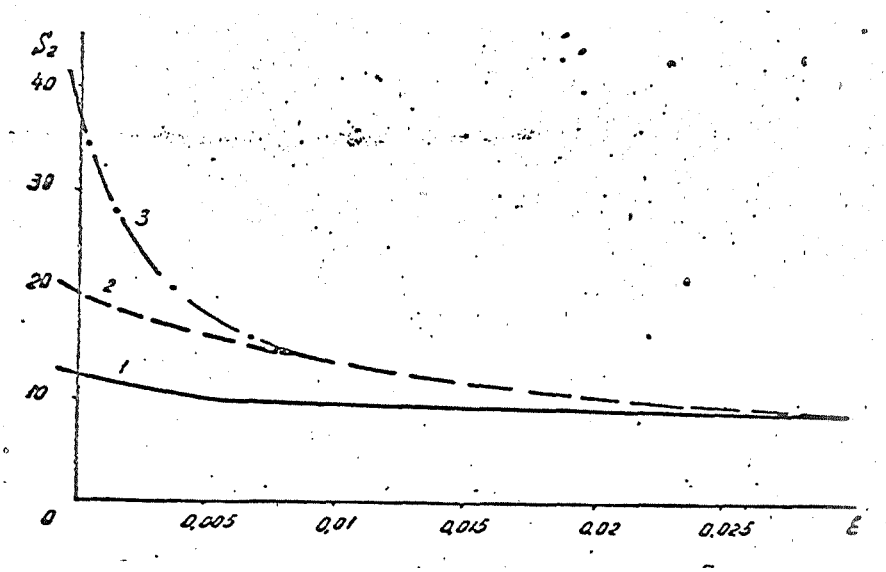

Figure 1. Areas of pore space, S_2, in terms of unit strain, ε.

this relationship for three models. The number on the curve in this figure, and on all subsequent ones, corresponds to the number of the model. The curves can be used to approximate the porosity, which is 7.5% for the first model, 8% for the second, and 8.2% for the third.

Eq. (1) and (2) were used to compute Young's modulus in terms of pressure for all three models. Figure 2a shows Young's modulus, E, as a function of pressure, p, for these models when $p_2/p = 1/10$. This modulus for the first model is higher at atmospheric pressure than it is for the other two. There is only a slight increase in the modulus for this model with increase in pressure. The modulus for the second model is lower than for the first at atmospheric pressure, but its increase with increase in pressure is greater. Finally, the modulus for the third model is even lower at atmospheric pressure, its increase with pressure at low pressures is very great, but slight at higher pressures. The third model, evidently, is that of the behavior of many rocks under high pressure conditions. The second model describes some cases of /396
the behavior of sedimentary rocks.

Figure 3a shows the curves for Young's modulus, E, in terms of uniform pressure, p, for a basalt specimen from bending experiments [2]. As will be seen, the nature of the change in Young's modulus for the rock, and for the three models is the same.

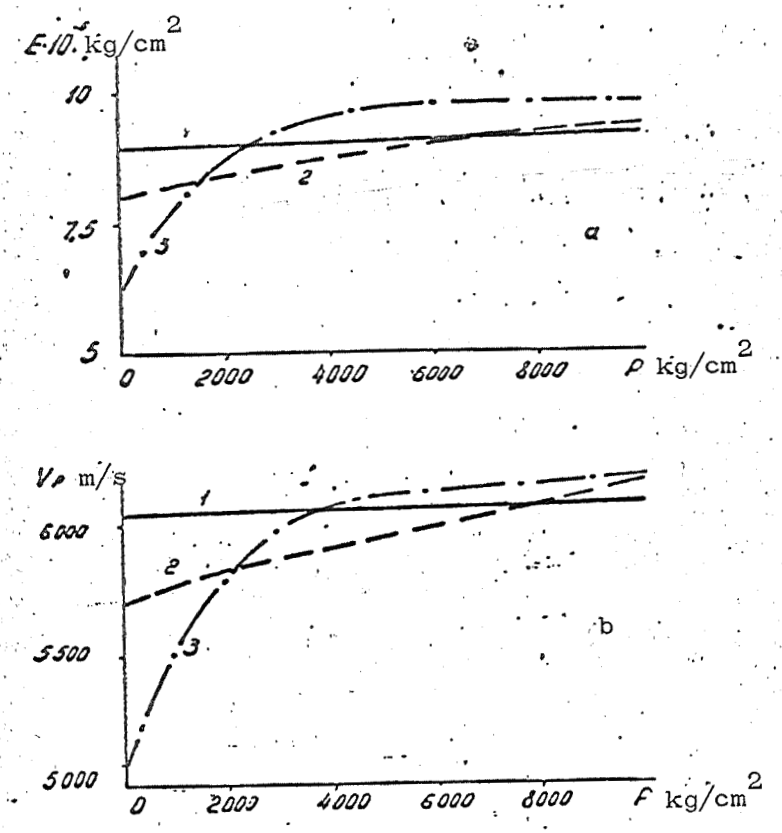

Figure 2. Young's modulus, E, and velocity of longitudinal waves propagation, v_p, in terms of uniform pressure p, when $p_2/p = 1/10$ for the models.

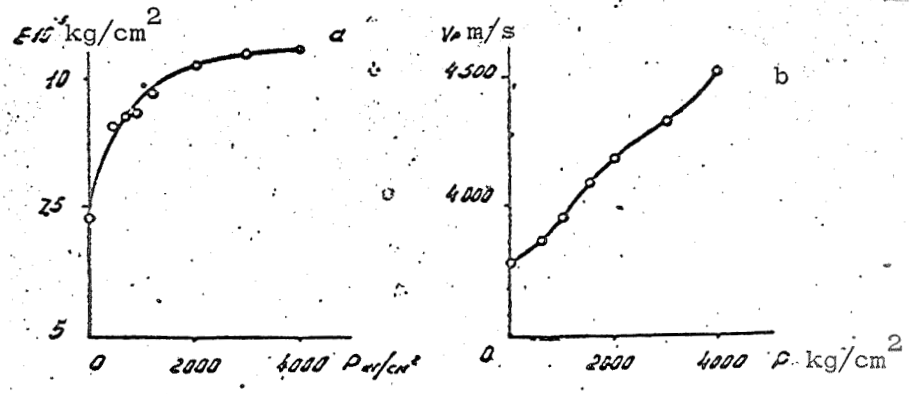

Figure 3. Young's modulus, E, in terms of uniform pressure, p, for basalt from bending experiments (a). Velocity of longitudinal waves, v_p, in terms of uniform pressure, p, for pelitomorphic limestone (b).

The velocity of the elastic waves can be computed by assigning a density of $\rho = 2.9$ g/cm^3, and using elasticity theory formulas. Figure 2b is a graphic of the velocity at which longitudinal waves are propagated in terms of uniform pressure for the models, and Figure 3b is a similar graphic for pelitomorphic limestone [5]. Eq. (2) can be used to find Poisson's ratio, and then, knowing two elastic characteristics, all the others can be found by using the elasticity theory formulas. /397

Let us see what effect uniform pressure has on the strength of solid bodies exhibiting porosity. Let us take it conditionally that the model will be destroyed by the action of normal stresses when the area of contacts in the model is, say, 60% of the total area. Figure 4 shows conditional breaking stresses, p_2 in terms of uniform pressure p. As we see, there is a sharp increase in strength with increase in pressure. At a pressure of p = 3,000 kg/cm^2, for example, the strength increases by a factor of almost 7. In the experiments with rocks, the bending strength increased by a factor of between 10 and 17 when the uniform pressure increase was to 3,000 kg/cm^2 [6]. Matsushima obtained this same large increase in strength in compression tests [7].

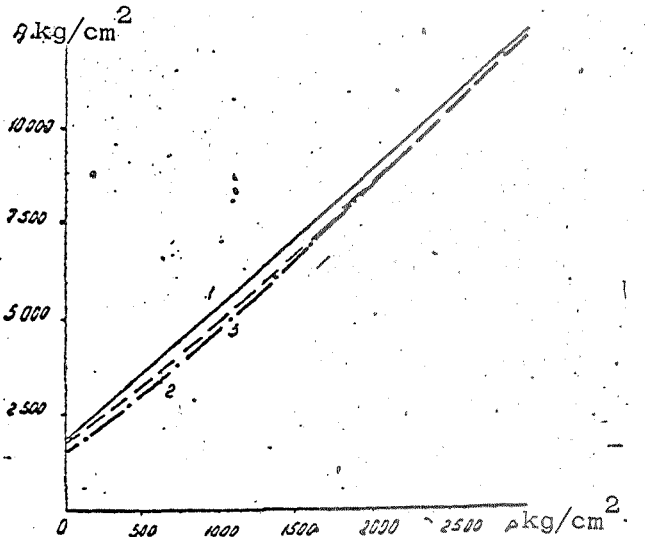

Figure 4. Breaking stress, p_2, in terms of uniform pressure, p, for models.

The conditional strength of model 3 under various uniform pressures will follow curve 3. The strength of models 1 and 2 will be higher than curves 1 and 2, respectively. Just how much curve 2 will be above curve 3 will depend on the rate at which the slots open up during loading. The same applies to curve 1. The only obvious point is that curve 2 will be above curve 3, and curve 1 will be above curve 2. All curves will coincide at high pressures. Real rocks can be destroyed at other than normal stresses. Tangential stresses will begin to destroy rocks as the uniform pressure is increased. The curves will be somewhat different in such case, of course.

Based on what has been said above, it can be pointed out that slotted and microslotted porosity have the greatest effect on change in the elastic parameters under high, uniform pressure conditions, and this is borne out by Figure 2. Knowing the relationship between the areas of contacts and the unit strain for the rock specimen, we can find the change in elastic characteristics under the effect of uniform pressure. And, conversely, we can find the relationship between the area of contacts and the unit strain, or pressure, by using Eq. (1) and the curve for elastic parameters in terms of pressure.

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